Thursday

- Any new comments from earlier topics
- Transverse linear errors and adjustments
  - Correction/adjustment of central trajectory
  - Gradient errors and adjustments
  - Chromaticity
- Edge Focusing

Steering Errors and Corrections

- Sources of steering errors
- Closed orbit distortions in circular accelerators
- Integer resonance in a synchrotron
- Trajectory adjustments

Effects of gradient errors

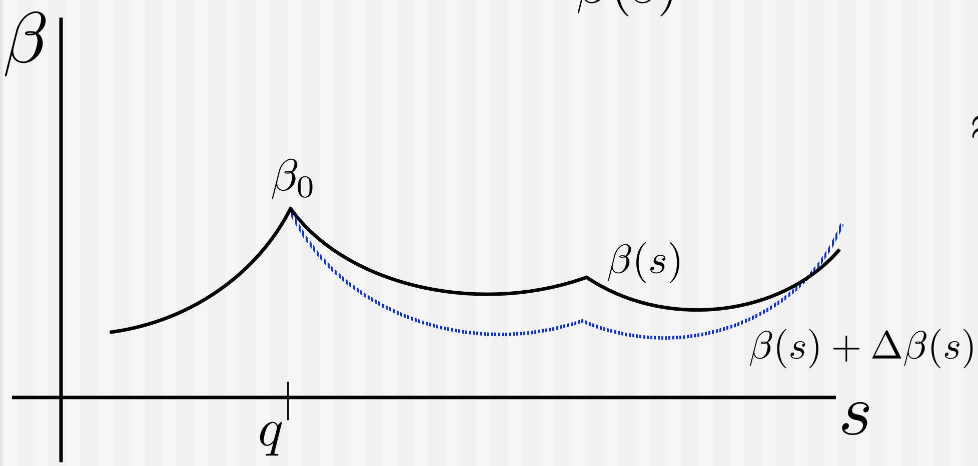
- Suppose gradient of a thin lens is changed by amount $\Delta B'$...

Here, we'll let $q \equiv \dfrac{\Delta B' \ell}{B\rho}$

At the source, $\Delta\alpha = \beta q$

Downstream of the source, $$\frac{\Delta\beta(s)}{\beta(s)} = -q\beta_0 \sin 2\Delta\psi_0 + \frac{1}{2}(q\beta_0)^2(1 - \cos 2\Delta\psi_0)$$

$$\approx -q\beta_0 \sin 2\Delta\psi_0$$ (for small $q\beta_0$)



Mismatch Invariant

- Consider two solutions to $\beta'' + 4K\beta = const.$ through a focusing system
  - for example, one may be the periodic solution, the other a perturbed solution
- Then,

propagate original solution

$$\begin{aligned}
J_{02} &= M J_{01} M^{-1} \\
J_{02} + \Delta J_2 &= M(J_{01} + \Delta J_1) M^{-1} \\
\Delta J_2 &= M \Delta J_1 M^{-1} \\
\det \Delta J_2 &= \det M \det \Delta J_1 \det M^{-1} \\
\det \Delta J_2 &= \det \Delta J_1
\end{aligned}$$

propagate perturbed solution

Thus, $\det \Delta J$ for two solutions is a constant along a beamline

Expressions for Determinant of ΔJ

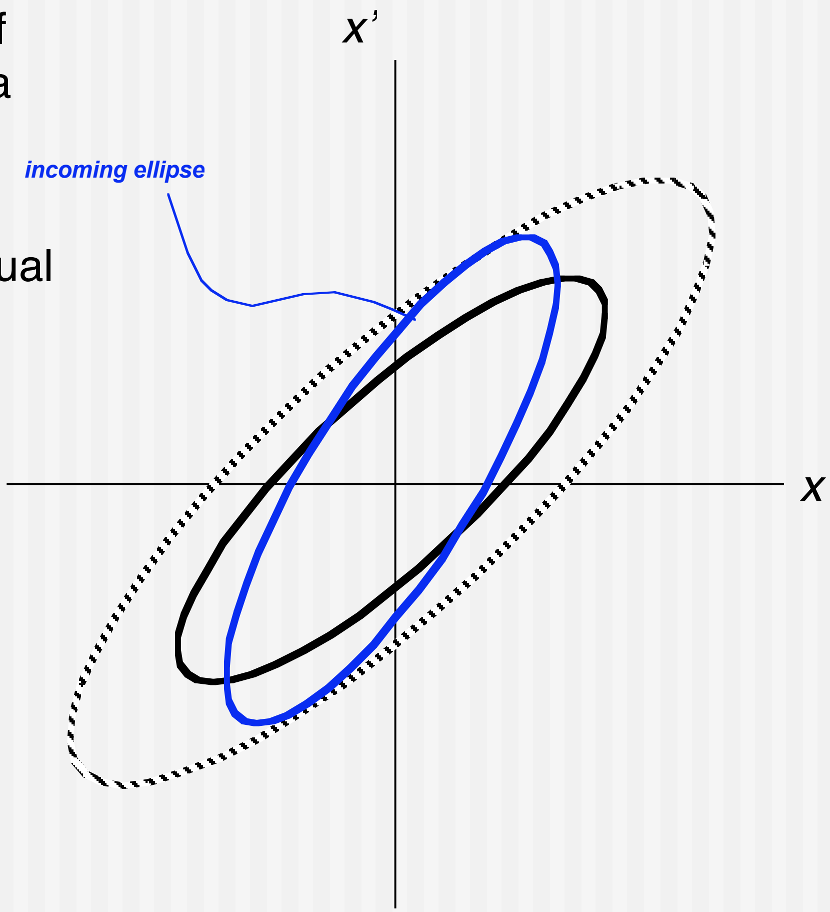
$$
\begin{aligned}
\det \Delta J &= \det(J_1 - J_0) \\
&= \begin{vmatrix} \Delta\alpha & \Delta\beta \\ -\Delta\gamma & -\Delta\alpha \end{vmatrix} \\
&= -\Delta\alpha^2 + \Delta\beta\Delta\gamma \\
&= 2 - (\beta_0\gamma_1 + \beta_1\gamma_0 - 2\alpha_0\alpha_1) \\
&= -\frac{\left(\frac{\Delta\beta}{\beta_0}\right)^2 + \left(\Delta\alpha - \alpha_0\frac{\Delta\beta}{\beta_0}\right)^2}{1 + \frac{\Delta\beta}{\beta_0}} < 0
\end{aligned}
$$

Some Examples...

- Injection Mismatch and Emittance Dilution
- Adjustment of Quadrupole in Beam Line
- Tune Shift in a Synchrotron
- Half-integer Stopband in a Synchrotron

Injection Mismatch and Emittance Dilution

- Suppose beam arrives through a transfer line into a synchrotron, but the beta function of the line is not matched to the periodic beta function of the ring...
- Particles will begin to follow phase space trajectories dictated by the ring lattice; actual nonlinearities of the real accelerator will cause their motion to decohere
- Net result: emittance dilution



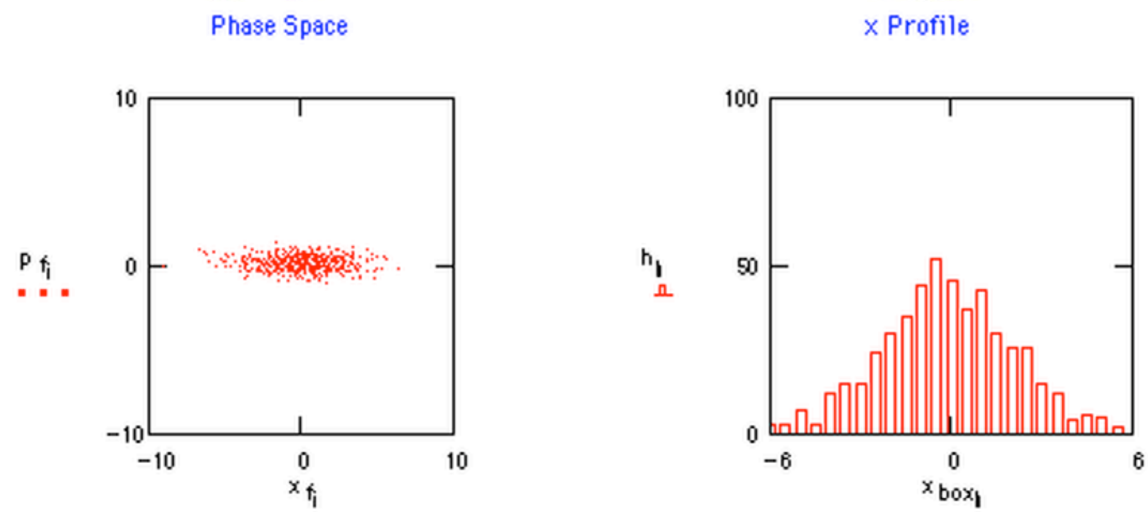
if $\epsilon \sim \langle x^2 \rangle$, then

$$\boxed{\epsilon/\epsilon_0 = 1 - \frac{1}{2}\det \Delta J}$$

Let’s Role the Video Tape...

Matched:
  circular
Mismatched:
  elliptical

Decoherence occurs in a “real” accelerator due to inherent non-linear fields; particle motion gets out of phase, results in an apparent increase of phase space area



$\mathrm{mean}(x_f) = 0.056$ $\mathrm{stdev}(x_f) = 2.283$

Emittance Increase: $\mathrm{stdev}(x_f)^2 = 5.21$

Predicted "typical" values:

(Steering Mismatch) $1 + \frac{1}{2}\cdot\Delta x^2 = 1$

FRAME = 0

(Amplitude function Mismatch) $1 + \frac{1}{2}\cdot\frac{\delta\beta^2}{1+\delta\beta} = 2.6$

Quad Error/Adjustment

- Suppose a beam line is perfectly matched to a circular accelerator downstream.
- Suppose now adjust a thin lens quad in the beam line, with nominal focal length *F*; just after the lens,
  $$\Delta\alpha = \beta_0 q = (\beta_0/F)(\Delta B'/B')$$
  $$\Delta\beta = 0$$
- The mismatch invariant, ΔJ, is constant downstream, so...
  - the beta function distortion through the rest of the line will have amplitude: $\Delta\beta/\beta \approx \beta_0 q$
  - at the source, and at injection point to the accelerator, $\Delta J = -\Delta\alpha^2$
  - and the resulting emittance growth will be

$$\boxed{\epsilon/\epsilon_0 = 1 + \frac{1}{2}(\beta_0/F)^2(\Delta I/I)^2}$$

Suppose $\beta_0 = 45$ m, $F = 15$ m, and a 5% change is made; then $\Delta\beta/\beta \approx 15\%$, but $\epsilon/\epsilon_0 \approx 1.01$

Tune Shift in a Synchrotron

- Insert “thin quad” at one point in the synchrotron:

$$M = M_q M_0 = \begin{pmatrix} 1 & 0 \\ -q & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c - aq & d - bq \end{pmatrix}$$

$$\Longrightarrow \quad \mathrm{tr} M = a + d - bq = \mathrm{tr} M_0 - q\, \beta_0 \sin 2\pi\nu_0$$

$$\cos 2\pi(\nu_0 + \Delta\nu) = \cos 2\pi\nu_0 - \frac{1}{2} q \beta_0 \sin 2\pi\nu_0$$

For small changes or small errors:

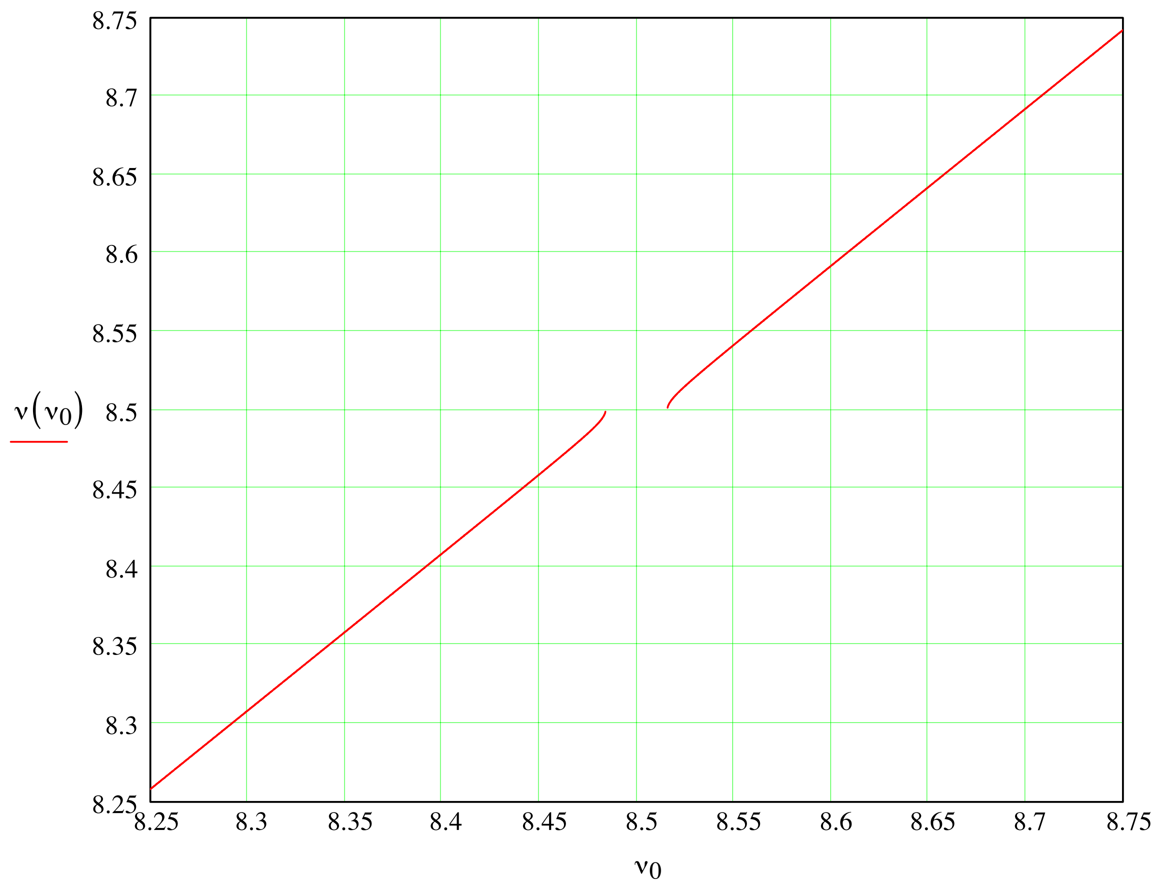
$$\boxed{\Delta\nu \approx \frac{1}{4\pi} \beta_0 q}$$

Note: will also generate a distortion of amplitude function...

Half-Integer Stopband

- Actual tune change due to gradient error:

$$\cos 2\pi\nu = \cos 2\pi\nu_0 - \frac{1}{2} q\beta_0 \sin 2\pi\nu_0$$



For given gradient error, or distribution of errors, as approach tune with half-integer value the lattice will become unstable -- the "stopband width" is the spread of unstable tune values.

Chromaticity

- Phase/tune vs. momentum
- Sources of chromaticity
  - The natural chromaticity of a synchrotron
    - chromaticity of a low-beta region
  - Sextupole field errors
- Adjustment, using sextupole magnets
- Influence of nonlinear fields on particle motion

Homework Due Friday

- Problem Set 4: #4, 5, and 6